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Granular Synthesis of Sounds through Fuzzyfied Markov Chains

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ABSTRACT
In this paper we introduce a new model for granular synthesis using Markov Chains and Fuzzy Sets. Whereas Markov Chains are used to control the evolution of the sound in time, Fuzzy Sets are employed to define the internal structure of the sound grains. It is shown also how the fuzzy structure of grains changes the markov Process. We provide the mathematical foundations of the model and briefly discuss how we have implemented it in a MATLAB program named Fuzzkov 1.0.

INTRODUCTION
Granular synthesis [6] is commonly known as a technique that works by generating a rapid succession of tiny sounds, metaphorically referred to as sound grains or yet as microsounds [7, 8]. Granular synthesis is widely used by musicians to compose electronic or computer music because it can produce a wide range of different sounds, but it also has been used in speech synthesis [4, 5]. Clearly a discussion about musical aesthetics arise from these developments and although it is a very interesting topic by itself we will not deal with these matters in this paper. A good account of the aesthetics of microsound can be found in reference [9].

Granular synthesis is largely based upon Dennis Gabor idea of representing a sound using hundreds or thousands of elementary sound particles [3].

In this work we take Curtis Roads' definition of sound grain as a point of departure to develop a formal but flexible granular synthesis model. The model uses stochastic processes, namely Markov Chains with Transition Probability Matrix modulated by Membership Functions of the grains with values in the interval $[0, 1]$, which gives the grains their fuzzy characteristics. Thus, we propose a new method for controlling the grains by intertwining Stochastic Processes and Fuzzy Set Theory, where the content of the grains (or internal variables) can change their transition probabilities between states. For the sake of clarity, we have chosen a very simple State Space to introduce the model, where each grain is a state of a Grain Vector $\mathbf{G}$. Therefore, the membership functions in this case modulate the transition probabilities between states (i.e., grains), changing their ordering position in the time domain. This paper introduced just one of several possible modes of interaction between internal and external control variables.

FUZZY GRAIN AND ITS MATRIX REPRESENTATION

Let us denote Ω the space of all possible oscillators, that is the *frequency* $\times$ *amplitude* space of the ordered pair (ω, a), where the variables ω and a varies in some suitable real intervals. Ω is referred to as a Parameters Space. We define, formally, a grain as a finite collection of points $\{(\omega_i(t), a_i(t)), i = 1, 2, \ldots, N\}$ in Ω, which is taken here as a state of a Markov Chain. A grain can be described by its Fourier Partials inside a real interval I. Its spectral content can be written, without loss of generality, as

$$G(t) = \sum_{n=1}^{N} a_n sin[2\pi\omega_n t + \delta_n], \qquad (1)$$

where a_n, ω_n, δ_n reads for amplitude, frequency and a possible phase, respectively.

In granular synthesis a sound can be viewed as a quick stream of grains which, from a geometrical point of view, describes a trajectory in the Ω space.

A grain g_i with r Fourier Partials can be read as a $2 \times r$ matrix:

$$g^i = \begin{bmatrix} \omega_1^i & a_1^i \\ \omega_2^i & a_2^i \\ \vdots & \vdots \\ \omega_r^i & a_r^i \end{bmatrix} \qquad (2)$$

Now, a fuzzy grain can be represented as a 3 column matrix

$$G^i = \left[\begin{array}{cc|c} \omega_1^i & a_1^i & \alpha_1^i \\ \omega_2^i & a_2^i & \alpha_2^i \\ \vdots & \vdots & \vdots \\ \omega_r^i & a_r^i & \alpha_r^i \end{array}\right] \qquad (3)$$

where we have introduced a third column with the membership frequency and amplitude values of each partial of the grain G^i. Note that g^i is a particular case of G^i for $\alpha_1^i = \alpha_2^i = \ldots = \alpha_r^i = 1$.

MARKOV PROCESSES FOR FUZZY GRAINS

Fuzzy sets, first proposed by Lofti Zadeh [11] are able for handling uncertainty, imprecisions or vagueness. Below we show how the membership functions of fuzzy grains can modify the Markov Transition Matrix and so we get a fuzzy control for the Markov Chain. For a good account of Fuzzy Sets the reader is refereed to [1, 2]. Let us consider a grain described by its Fourier-like equation (1). Each subset of points in Ω represents a grain with particular Fourier partials, that is, it is a sum of basic sinusoidal frequencies.

With the above defined matrices G^i, it is possible to define an unambiguously time evolution of grains through out Markov Chains. This is usually accomplished through a Fuzzy Transition Table, constructed as follows: firstly, suppose that we have a transition matrix for ordinary grains, that is, with no membership vector yet defined. This can be written as follows:

$$\left[\begin{array}{l|cccc} & g^1 & g^2 & \ldots & g^N \\ \hline g^1 & p^{11} & p^{12} & \ldots & p^{1N} \\ g^2 & p^{21} & p^{22} & \ldots & p^{2N} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ g^N & p^{1N} & p^{2N} & \ldots & p^{NN} \end{array}\right] \qquad (4)$$

which can be viewed as a function

$$\begin{array}{rccl} p: & \mathbf{g} \times \mathbf{g} & \longrightarrow & [0,1] \\ & (g^i, g^j) & \longmapsto & p\left(g^i, g^j\right) = p^{ij} \end{array}$$

Now, we define a Fuzzy Extended Probability Transition Matrix (or simply Fuzzy Transition Matrix) Q : $\mathbf{G} \times \mathbf{G} \longrightarrow [0,1]$ as

$$Q^{ij} = Q\left(G^i, G^j\right) = \Phi^{ij} * p^{ij} \qquad (5)$$

where the symbol $*$ means a matrix operation (e.g., a scalar product, a matrix product or any other well defined operation). The function Φ^{ij} is generated as a finite number of applications of the following basic operations of fuzzy sets: for $i, j = 1, 2, \ldots, N$, we define

1. $$\phi^{ij} = \max_{1 \le k \le r} \left\{\alpha_k^i, \alpha_k^j\right\}, \qquad (6)$$

   where α^i and α^j are the membership vectors of the grains G^i and G^j respectively.

2. $$\phi^{ij} = \min_{1 \le k \le r} \left\{\alpha_k^i, \alpha_k^j\right\}, \qquad (7)$$

   where α^i and α^j are the membership vector of the grains G^i and G^j respectively.

3. $$\alpha_c^i = 1 - \alpha^i. \qquad (8)$$

These result in a product like $\Phi^{ij} = \phi_1^{ij}\phi_2^{ij}\ldots\phi_l^{ij}$, where the third operation above can be performed on any product of α_i vectors. These are basic operations on Fuzzy Sets. See Diamond and Kloeden [1] for a introduction to Fuzzy Sets and their metrics. Note that since the membership function modulates the probability values p^{ij}, the condition for the probability sum $\sum_{j=1}^{N} Q^{ij} = 1$ can be violated. In order to solve this problem we renormalize the matrix Q^{ij} as follows. Denoting $q_i = \sum_{k=1}^{N} Q_{ik}$ we define the elements of matrix $\mathbf{P}$ as

$$P_{ij} = Q^{ij}/q^i \qquad i, j = 1, 2, \ldots, N \qquad (9)$$

Now the probability property $\sum_{j=1}^{N} P_{ij} = 1$ is clearly satisfied. The above definition shows that the internal fuzzy content of the grains have a weight (through the function Φ^{ij}) for their transition to a next state of the Markov Chain.

The Fuzzy Transition Matrix (or Table) now reads

$$\left[\begin{array}{c|cccc} & G^1 & G^2 & \dots & G^N \\ \hline G^1 & P^{11} & P^{12} & \dots & P^{1N} \\ G^2 & P^{21} & P^{22} & \dots & P^{2N} \\ \dots & \dots & \dots & \dots & \dots \\ G^N & P^{N1} & P^{N2} & \dots & P^{NN} \end{array}\right] \quad (10)$$

In this simple model a transition from one state to another corresponds to a jump from a particular grain to another in the grain vector **G**. In adition the fuzzy content of a grain, that is, its membership vector, can have a significant weight on the probability transition. Since the process is finite, a criterium to halt the process is needed here. This will be discussed in the next section.

The above model is suitable for several kinds of matrix operations on internal as well external variables controlling the grains behaviour in time. There is plenty of room for the definition of a great number of different methods to generate and control the grains. We present one of such methods below.

CONTROL OF GRAIN STREAMS

There exist many different ways (algorithms) to control the evolution of the grains in time. We show here one by using the so called Hausdorf Metric which is suitable to measure distance between sets (grains are finite and discrete subsets of Ω). Its definition is as follows [1].

Suppose that the space $\Omega = \mathbb{R}^{2N}$ has a metric $d(x, y)$. Let x be a point in Ω and A a nonempty subset of Ω. We define the distance of the point x to the set A as:

$$\delta(x, A) = inf\,\{d(x, y), y \in A\}\,. \quad (11)$$

The Hausdorff separation of a set B from a set A is defined by

$$\Delta(B, A) = sup\,\{d(y, A), y \in B\} \quad (12)$$

In general, Δ is not symmetric, that is $\Delta(A, B) \neq \Delta(B, A)$. In order to get a symmetric one we define the so called Hausdorff distance by

$$d_H\,(A, B) = max\,\{\Delta(A, B), \Delta(B, A)\} \quad (13)$$

With this distance function (Ω, d_H) is a Metric Space.

Time evolution can be better controlled using a fuzzy metric that takes into account the degree of membership of the Fourier partials inside each grain. In other words, partials with low membership coefficients contribute little for the Hausdorff distance measure between the grains. Membership vectors define the fuzzy character of the grains, or in a musical jargon, their weighted harmonic content. A metric control is closely related to the notions of approximation and/or the maximal time (or number of steps) available to run a process. Below we indicate three stop criteria we devised to halt a Markov Chain in our model of granular synthesis.

Halting Criteria

1. *Convergent Type*: If the distance between the last generated grain and a fixed grain (target) is smaller than a prefixed arbitrary number ϵ, the process halts.
2. *Cauchy Type*: If the distance between two states is smaller than ϵ the process halts.
3. *Maximal Number of Steps Type (MNS)*: Fix the maximum number of steps for the process to halt.

Any of the above criteria can be used to halt the process. Of course *Maximal Number of Steps Type* is the simplest one, since no metric is required. In our program *Fuzzkov 1.0* we have implemented fully the MNS and partially, the Cauchy type, at the Hausdorff Metric level, but not at the Fuzzy Metric level. We have implemented the Hausdorff Metric as an inequality, so that FuzzKov 1.0 runs in loops until it is satisfied. We obtained good results for both controls of the grains streams working together.

In adition we can also specify a number of different formal settings to update the internal content of the grains at each step of the Markov Chain. This provides the means to control the evolution of the macrosound in the Ω Space. Below we indicate two updating methods.

Grains Updating

1. *No Updating: no change in the internal content*

   In this case, each grain G_i corresponds to a state of the grain vector **G** and no operation is applied to the internal structure of the grains. Nevertheless this procedure takes into account the fuzzy nature of grains as the role of the membership vectors is to produce new arrangements in time (that is, permutations) for the prefixed grains.
2. *Core Merged Grains*

   In this case we update l-th step grain as a subset merging l previous grains of the Markov Chain; e.g., $G^0, G^1, \dots, G^{l-1}$. For the sake of clarity, we ignored all the other subindexes. Define the *l-Mean Frequency* as

   $$\overline{\omega}^{(l)} = \sum_{k=1}^{r} \frac{\omega_k^0 + \omega_k^1 + \ldots + \omega_k^{l-1}}{l} \quad (14)$$

   and take the r closest frequencies from the set $U_l = \bigcup_{k=0}^{l-1} G^k$ to the mean frequency $\overline{\omega}^{(l)}$ and update G^l (with the same letter) as

   $$G^l = [(\omega_{k_1}, a_{k_1}), (\omega_{k_2}, a_{k_2}), \ldots, (\omega_{k_r}, a_{k_r})]\,.$$

This procedure leads to a concentration of frequencies within a narrow bandwidth, but with a large bandwidth for the amplitudes. The *halting criterion* here can be taken as the *Cauchy type*. Given an arbitrary (but small) number ϵ, the process stops if $d_H(G_i, G_{i+1}) \leq \epsilon$, where the distance between two points used for defining the above Hausdorf Distance is given by, for example:

$$d((\omega_i, a_i), (\omega_j, a_j)) = \max_{1 \leq k \leq r} |\omega_i - \omega_j|. \qquad (15)$$

If we fix a particular grain in the Ω space, such as $\overline{G}$, we can consider the *Convergent halt criterion* , that is the proceess stops if $d_H(G_i, \overline{G}) \leq \epsilon$.

We can also take the *mean frequency* only of the last m grains and so it reads as

$$\overline{\omega}^{(l)} = \sum_{k=1}^{r} \frac{\omega_k^{l-m} + \omega_k^{l-m+1} + \ldots + \omega_k^{l-1}}{m} \qquad (16)$$

and take the r closest frequencies to $\overline{\omega}^{(l)}$ from the set $U_l = \bigcup_{k=l-m}^{l-1} G^k$. Clearly, for $m = l$ we get the previous model.

A short description of the Diagram of Fuzzkov 1.0 (Fig 1) is as follows. The grains are generated by randomly (uniform and gaussian) 3-dimensional matrices A with dimensions $2 \times r \times N$ which include r normalized frequencies and amplitudes for N grains (Fourier Partials). We have taken the uniform as well the Gaussian distribution of probability to gerated them. From this we get a Matrix $B(2, 1, N)$ with the sum of Fourier Partials for the N grains. A Markov transition Matrix $p(N, N)$ is generated and modified by a Membership Matrix $Memb(N, N)$. A number of diferent operations are available to do this modification. So we get a fuzzyfied Markov Matrix $Q(N, N)$ which operates on an aaray of probabilities vectors $u(n+1, N)$. Next, a particular filter choose the index of the maximal value of each probability vector $I(1, n+1)$. Finally, the program reorder the Grain matrix $B(2, 1, N)$ along the index vector $I(1, n+1)$ and produce the sound as well other outputs for analysis.

EXAMPLES AND ANALYSIS OF THE RESULTS

We have implemented a prototype of our model using Matlab in which Membership Matrices modulate a Transition Probability Matrix of a Markov Chain, but the internal content of the grains are not changed during the Process. Thus, our model can be thought of as a *Coarse Grain Fuzzy Synthesis*. We have used the MNS and Cauchy criteria, by using the Hausdorff Metric on the Grain Space, in order to halt the process. Weakly convergent process have lead to rich varities of timbre along sound streams. This is because the system has time enough to explore the possibilities during the Markov Process. We have in adition included some special effects commonly used in granular syntehsis such as modulation through time windows for the grains, which avoids glitches and, in the macro scale, we included crescendo and decrescendo effects. In fact this can be done more generally by using an modulation function with an arbitary number of peaks and regions with increasing as well decreasing rates.

We have normalized all the sound signals, so they are more suitable to analysis and comparison. After record the digital signal as a *wav.file* the program has three outpus: *sound stream, spectrogram* and *plot of the probability vector evolution* from MATLAB. In adition,in order to analyse in a easy way our results, we used the WAVELAB program to get a *3D-analysis* in a time × frequency space.

CONCLUSION

We have presented a model for granular synthesis as a Markov Chain in which each grain is a possible state of a Grain Vector **G**. A major feature of our model is that the spectral components of the grains are coupled with the state transition probability through grain's membership vectors. This allows the user more flexibility in a higher level as well more variability to control the sequence of grains of the Markov Chain. We have implemented a computer algorithm named *Fuzzkov 1.0*, written in MATLAB, in which the membership functions modulate the Transition Probability Matrix. Nevertheless the internal contents of the grains are not changed by them in this first version of Fuzzkov. In this way, the present model can be understood as a *Coarse Grain Fuzzy Synthesis*. A more complex program should allow the use of Fuzzy Functions to emphasize some particular components of the spectral content of the grains and then to drive the sound flow by updating them at each step through mergin and selecting the most representative frequencies and amplitudes to construct the next grain of the stream. As Halt Criteria we used the Maximal Number of Steps, as well the Cauchy type with the Hausdorff Metric between grains. A model in which the states of the Markov Chain are related to grains's subsets (*Fine Grain*), as well an effective use of Fuzzy Metrics, will be presented elsewhere.

A distintive point of our approach is its flexibility of macro manipulation by fuzzy matrices parameters. At present time we can consider this control as a toy model. Nevertheless it has a great potential to include new aspects of the fuzzy approach. Also all effects are included in the algorithm and so no external device is needed. Our experiments with *Fuzzkov 1.0* has shown that its audio output is comparable with the most recent granular synthesizers and in adition, depending on the inputs parameters it can provide surprisingly new sounds

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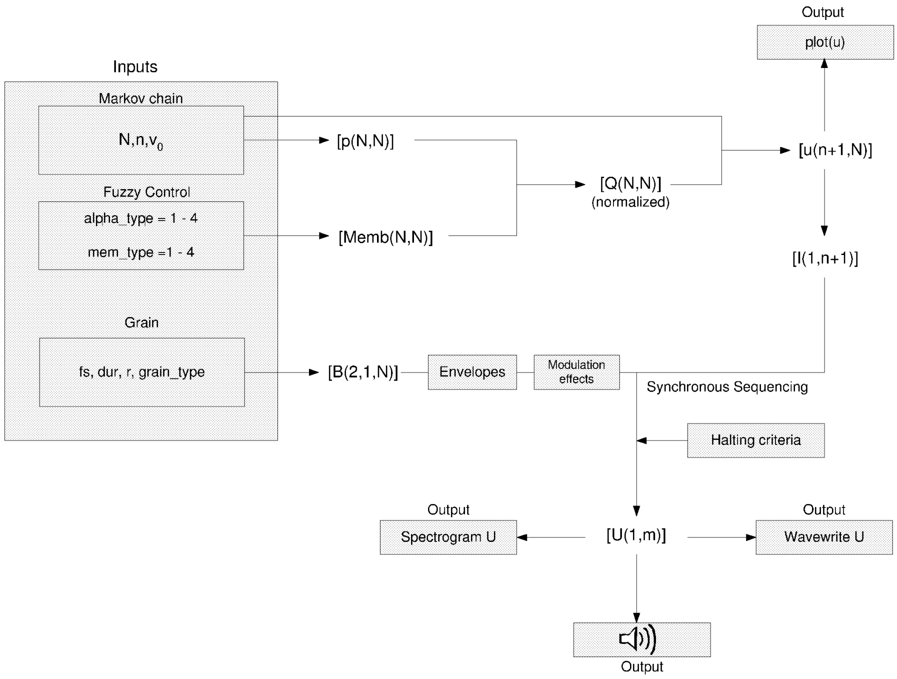

Fig. 1: Diagram of *FuzzKov 1.0*

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