Interactive Intelligent Systems Workshop: Music Constraint Programming (4)

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Outline

Why Discussing the Search Process?

- Music constraint programming greatly simplifies the implementation of complex music theory models
  - User only specifies all constraints the solution should fulfil
  - A constraint solver finds solution(s) for a constraint satisfaction problem (CSP) via search
- However, reasonably efficient search vital to make system useful in practice
- This talk discusses how musical CSPs are solved efficiently

Variable and Value Orderings I

Variable ordering

- Order in which variables are visited during the search process
- Variable ordering has great impact on efficiency (size of resulting search tree)
- Suitable variable ordering highly problem dependent

Static vs. dynamic variable ordering

Variable ordering is either fixed before the search starts (static), or computed during the search process (dynamic)

Variable and Value Orderings II

First-fail principle

- Common principle for designing dynamic variable orderings
- Essence: deal with hard cases first – if failure is inevitable, better fail early
- Typical approaches: first visit variable with smallest domain, or variable with most constraints applied to it

Variable and Value Orderings III

Value ordering

- Order in which variable domain values are considered during the search process (speculative computation: values may fail and others may be tried later)
- Has impact on efficiency, but also on quality of first solution
- Common principle: succeed-first principle or best-first heuristic
- Example for musical CSP: good heuristic is often a randomised domain value selection (avoids uniformness)

'Variable orderings' in manual composition I

- In classical music education, the harmonic structure (underlying chord progression) often written before the actual note pitches
- Some contemporary composers finish rhythmical structure and aspects of instrumentation before writing note pitches
- Melody plus accompany setting: melody is often written first, and then the accompaniment
- Homophonic music: notes of outer voices (sopran and bass) are usually written before middle voices
- Contrapuctual music: composer usually progresses with all voices more or less in parallel

'Variable orderings' in manual composition II

These observations suggest: variable orderings also play an important role for efficiently/adequately solving musical CSPs

Variable Orderings for Musial CSPs in Existing Systems

Existing constraint systems support a single and static variable ordering: optimised for specific class of musical CSPs – but less suitable for others

- Many music constaint systems represent music simply as a sequence of score objects (e.g., Situation, PWConstraints subsystem PMC – two seminal systems)
- The static variable ordering visits the variables in the order of the sequence

Left-to-Right Variable Ordering I

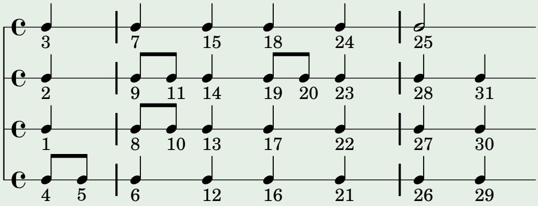
Left-to-right variable ordering

Static variable ordering of Score-PMC, a subsystem of PWConstraints for polyphonic music [Laurson, 1996]

- Visit note with smaller start time more early
- If two notes share the same start time
  - Visit note of lower voice before note of upper voice
  - Visit longer note before shorter note

Left-to-Right Variable Ordering II

Left-to-right variable ordering, demonstrated at a Bach chorale (cf. [Laurson, 1996])



Left-to-Right Variable Ordering III

Advantages

- Efficient solving of polyphonic CSP
- Rhythmical structure can be arbitrarily complex

Disadvantages

- Rhythmical structure must be fully determined in CSP definition (!)
- This variable ordering hard-wired in Score-PMC: less efficient for, e.g., harmonic CSPs with complex constraints on underlying harmonic structure (causes redundant work at note pitches)

Motivation

We want to solve various different musical CSPs: harmonic, contrapuctual etc.
Therefore, we want to choose a variable ordering suitable for the CSP at hand

The following section introduces a constraint programming model which supports dynamic and user-definable variable and value orderings

Message-Passing Concurrency: the Underlying Programming Model I

- Partial values (logic variables): variable can be
    - free (nothing is known about its value)
    - partially determined (e.g. it is a list with undetermined elements)
    - fully determined
    - Constraints add information about variable values (e.g., unification, numeric constraints)
- Concurrency: computations executed in multiple *threads* (created explicitly)

Message-Passing Concurrency: the Underlying Programming Model II

- Synchronisation of threads on variables:
  - Thread *blocks* if logic variables used in a statement of the thread lack required information
  - Another thread might provide this information – threads communicate via (dataflow) variables
- First-class procedures: procedures (abstracting computations) are first-class values, and support lexical scope
- Ports: communication channel for sending data between concurrent threads, including many-to-one communication (asynchronous FIFO)

Message-Passing Concurrency: the Underlying Programming Model III

Note

- Message-passing concurrency is highly expressive programming model: it greatly simplifies writing concurrent programs with massive number of threads
- Reason: stateless concurrency (no conflicts of shared resources can occur)

Example: Erlang programming language

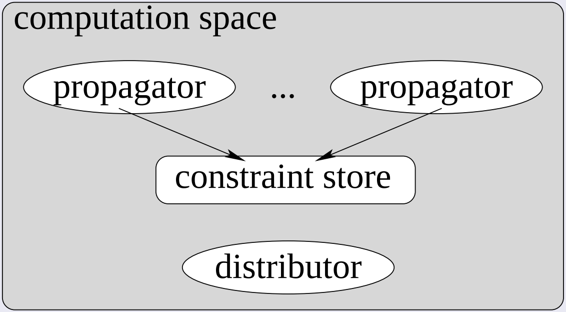
Model of programming language *Erlang* is similar to this message-passing concurrency model. Ericsson uses Erlang successfully in several Ericsson products for telecommunication

From Message-Passing Concurrency to Constraint Programming

- No support for search in message-passing concurrent model
- Adding *computational spaces* provides support for speculative computations and search
- In spaced-based constraint model, search is encapulated
- Alternative to *backtracking*-based search (as in Prolog) – backtracking not feasible with concurrency and interoperating with external world
- We only study simplified view on the constraint model based on spaces

The Computation Space

Propagate and search: a *computation space* encapsulates information available on a CSP at a certain stage during the search process



The Constraint Store

- Constraint store: stores information on variable values – conjunction of basic constraints
- Basic constraint: representation of information on partial value of a single variable. Example for finite domain integers (FD ints): two forms possible
  - $X \in \mathbf{D}$ means $\mathbf{D}$ (a set of natural numbers) is *domain* of X, special case $X \in \{n\}$ means $X = n$ (X determined to n)
  - $X = Y$ means X and Y are equal (unified) – both can be undetermined

Example constraint store

$$X \in \{1, \ldots, 5\} \wedge Y = 7 \wedge Z = X$$

Constraint Propagation I

Propagator

- Any more complex constraint (non-basic constraint) expressed by propagator
- A propagator is a concurrent agent
- Propagator aims to add information (i.e. narrows variable domains) which is
  - consistent with constraint store
  - follows from constraint expressed by propagator
- Implemented by algorithm usually highly optimised for its specific constraint

Constraint Propagation II

Example: propagator $X < Y$ narrows domain of X and Y

- Store before propagation: $X \in \{1, \ldots, 5\} \wedge Y \in \{1, \ldots, 5\}$
- Store after propagation: $X \in \{1, \ldots, 4\} \wedge Y \in \{2, \ldots, 5\}$

Constraint Propagation III

Note

Constraint propagation does not necessarily lead to a solution

Example: propagators $X \neq Y$, $X \neq Z$, and $Y \neq Z$ cannot reduce the domains further

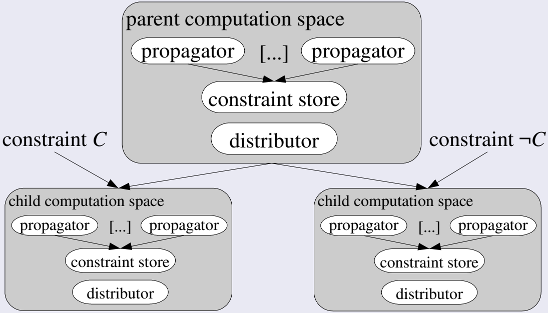
$$X \in \{1,2\} \wedge Y \in \{1,2\} \wedge Z \in \{1,2\}$$

Stable space

No further propagation is possible: hosting computation space is *stable*

Constraint Distribution I

Constraint distribution creates two child spaces which are the result of two complementary decisions (expressed by the two added constraints C and $\neg C$)



Constraint Distribution II

- Constraint distribution (branching): proceeds to spaces easier to solve, but with same solution set (search)
- Distributor: concurrent agent
    - Waits until space is stable
    - Then creates two child spaces (copies of parent space)
    - Add some basic constraint C to store of one child space and its complement $\neg C$ to store of other child space
    - Important: choose such C and $\neg C$ which trigger further constraint propagation

Constraint Distribution III

Example distribution strategy: first-fail

Select variable with smallest domain, and determine it to its left-most domain value

Combination of constraint propagation and distribution is a complete search method for solving CSPs

First-Fail Distribution: a Musical Example I

CSP all-distance series definition (length 4)

$Xs :=$ list of 4 FD ints, each with domain $\{0, \ldots, 3\}$

$Dxs :=$ list of 3 FD ints, each with domain $\{1, \ldots, 3\}$

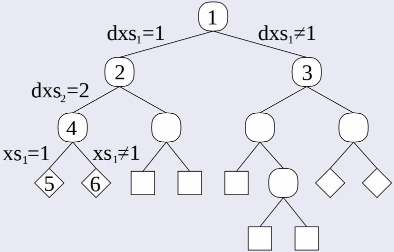
$$\bigwedge_{i=1}^{3} Dxs_i = |Xs_i - Xs_{i+1}|$$

$$\wedge\ distinct(Xs)$$

$$\wedge\ distinct(Dxs)$$

First-Fail Distribution: a Musical Example II

All-distance series: search tree for all solutions of length 4



Space	Domains of elements in *Xs*	Domains of elements in *Dxs*
1	$[\{0,\ldots,3\},\ldots,\{0,\ldots,3\}]$	$[\{1,\ldots,3\},\{1,\ldots,3\},\{1,\ldots,3\}]$
2	[{0, . . . , 3}, . . . , {0, . . . , 3}]	[1, {2, 3}, {2, 3}]
3	[{0, . . . , 3}, . . . , {0, . . . , 3}]	[{2, 3}, {1, 2, 3}, {1, 2, 3}]
4	[{1, 2}, {1, 2}, {0, 3}, {0, 3}]	[1, 2, 3]
5	[1, 2, 0, 3]	[1, 2, 3]
6	[2, 1, 3, 0]	[1, 2, 3]

Distribution Strategy Definition I

- Distribution strategy can be defined ‘from scratch’ (cf. [Schulte, 2002])
- More convenient: definition with a higher-level interface
- Simple interface example expects two first-class functions as arguments (see next slide)

Distribution Strategy Definition II

Order: which variable is distributed (variable ordering)

Boolean function expecting two variables. Returns *true* if first variable should be visited before the second

Value: how does distribution strategy effect domain of selected variable (value ordering)

Function expecting a variable, and returning a reduced domain specification for this variable (usually a single domain value)

Example: first-fail distribution strategy definition

Order: $myOrder(X, Y) := getDomSize(X) \leq getDomSize(Y)$
Value: $myValue(X) := getMinDomValue(X)$

Variable and Value Orderings

- User can freely define distribution strategies
- Defining a distribution strategy means defining shape of search tree: i.e., a variable and value ordering
- Next distribution step is always decided only when it is required: dynamic ordering
- Distribution strategies can be changed independently of problem definition

Principles for Efficient Distribution Design

- An efficient distribution strategy results in a relatively small search tree (little amount of failure)
- Constraint propagation never causes a fail (no redundant work)
- An efficient distribution strategy keeps distribution steps at minimum, i.e. helps constraint propagation to do most of the work
- Common example: first-fail principle (see above)

Resolve-Inaccessible-Context Principle I

Inaccessible score context

Set of score object which can not be accessed because of undetermined information
Example: if the rhythmical structure is undetermined, then the contexts of simultaneous notes are inaccessible

Note

- If inaccessible contexts are constrained, then constraints applied to inaccessible contexts can not propagate
- This occurs frequently in musical CSPs

Resolve-Inaccessible-Context Principle II

Resolve-inaccessible-context principle

- Resolve constrained inaccessible score contexts early in the search process
- A rule of thumb for designing score variable orderings, like first-fail principle

Other Features of the Space-Based Constraint Model

Besides propagation and distribution, the constraint model has more features – at least mentioned here

- Constraint propagation between variables with specific domains
- User-definable distribution strategy (branching strategies): specifies search tree
- User-definable exploration strategy: exploration of search tree
- Reified constraints: constraining the truth value of other constraints (e.g., with logical connnectives)
- Recomputation: trades memory for run time
- Parallel search: distribute workload of solver on multiple computers

Implementations of the Space-Based Constraint Model

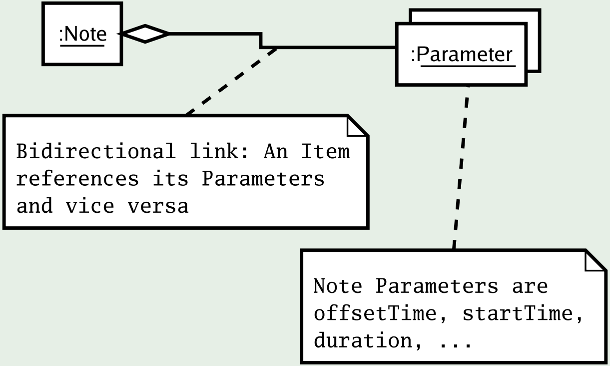
- Mozart: implementation of the multi-paradigm programming language Oz, `http://www.mozart-oz.org/`
- Gecode: C++ library, `http://www.gecode.org/`

Score Distribution Strategies

- Distribution strategies usually distribute plain variables
- Instead, score distribution strategies distribute parameter objects of music representation
- Advantage:
  - Parameter objects provide access to the score object they belong to, and that way to all information in score (via bidirectional links between score objects)
  - So, a score distribution can make an informed decision

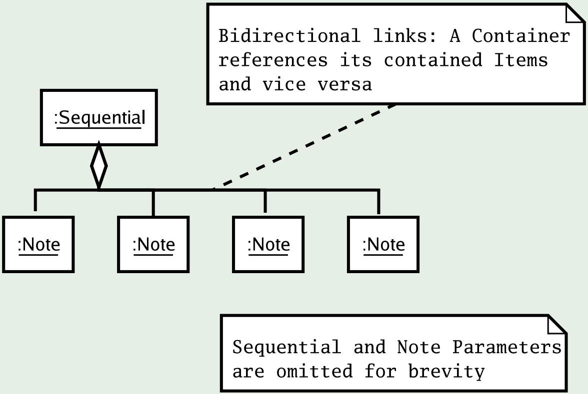
Recap: Bidirectional Links Between Score Objects I

The hierarchic structure of a single note and its contained parameters (UML)



Recap: Bidirectional Links Between Score Objects II

The hierarchic structure of a container with several contained notes



Definition: First-Fail Score Distribution Strategy

Recap: idea of first-fail distribution

Select parameter which stores the variable with smallest domain, and determine the variable to its left-most domain value

First-fail distribution strategy distributing parameters

Order:

$$myOrder(par_1, par_2) :=$$
$$getDomSize(getValue(par_1)) \leq getDomSize(getValue(par_2))$$

Value: $myValue(X) := getMinDomValue(X)$

Application: First-Fail Distribution Strategy I

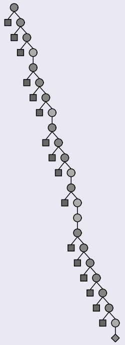
Musical example: Fuxian first-species counterpoint

`http://strasheela.sourceforge.net/strasheela/doc/Example-FuxianFirstSpeciesCounterpoint.html`

- All constraints can be applied directly (i.e. no inaccessible contexts in CSP definition)
- This makes it possible to apply an established general distribution strategy: first-fail

Application: First-Fail Distribution Strategy II

Search tree of Fuxian first-species counterpoint example



- Small search tree with only few failed notes (squares) until first solution is found (diamond) – i.e. constraint propagation does most of the work
- Runtime: ca. 50 msecs[a]

[a] Pentium 4, 3.2 GHz, 512 MB RAM, Linux Fedora Core 3

Resolving a Single Contexts I

- Typical example of inaccessible score context: if rhythmical structure is undetermined, then simultaneous notes are inaccessible
- One solution: determine all temporal parameters, before other parameters

Variable ordering which determines all temporal parameters first

Order: $myOrder(par_1, par_2) := isTemporalParameter(par_1)$

Resolving a Single Contexts II

- Variable ordering which first determines temporal parameters is only example
- Other example is harmonic CSP: explicitly represented analytical harmonic information should be determined before actual note pitches

Resolving Multiple Contexts in Order I

A variable ordering for harmonic CSPs

First determine the temporal structure, then the harmonic structure, and finally the actual note parameters in the order pitch class, octave, pitch

Order: $determineInOrder([isTemporalParameter,$
$isChordParameter,$
$isPitchClass]),$
$isPitchOctave]),$
$isPitch])$

Resolving Multiple Contexts in Order II

Function *determineInOrder* returns ordering function g

$determineInOrder(tests) :=$

let */* Append a default test function which always returns true.*

$allTests := append(tests, [f : f(x) := \ true])$

in $g : g(p_1, p_2) :=$

$getTestIndex(p_1, allTests) \leq getTestIndex(p_2, allTests)$

- Function *determineInOrder* expects list of boolean functions; returns variable ordering function which determines parameters in the order specified by the list of boolean functions
- Function *getTestIndex* expects object and list of boolean functions; returns index of first function returning true for given object

Application: Resolving Inaccessible Score Context I

Musical example: chord progression

`http://strasheela.sourceforge.net/strasheela/doc/Example-MicrotonalChordProgression.html`

`http://strasheela.sourceforge.net/strasheela/doc/Example-HarmonisedLindenmayerSystem.html`

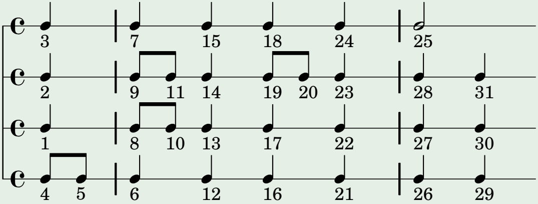
- Distribution strategy for CSP actually combines resolving of multiple score contexts with first-fail principle
- Example: in case of multiple chord parameters, the parameter with smallest domain is determined first

Left-to-Right Variable Ordering I

- Dynamic ordering version of variable ordering of Score-PMC (see above)
- Resolves inaccessible score context of simultaneous score objects dynamically
- Therefore, applicable for polyphonic CSP even when the rhythmical structure is undetermined in CSP definition

Left-to-Right Variable Ordering II

Recap: left-to-right variable ordering of Score-PMC



Definition: Left-to-Right Variable Ordering

A left-to-right dynamic variable ordering

Order: $myOrder(p_1, p_2) :=$

$$
\begin{array}{l}
\textbf{let}\ start_1 := getStartTime(getItem(p_1)) \\
\quad start_2 := getStartTime(getItem(p_2)) \\
\quad isStart_1Bound := (getDomSize(start_1) = 1) \\
\textbf{in}\ \textbf{if}\ isStart_1Bound \wedge (getDomSize(start_2) = 1) \\
\quad \textbf{then}\ \textbf{if}\ start_1 = start_2 \\
\qquad \textbf{then}\ isTemporalParameter(p_1) \\
\qquad \textbf{else}\ start_1 \leq start_2 \\
\quad \textbf{else}\ isStart_1Bound
\end{array}
$$

Application: Left-to-Right Variable Ordering I

Musical example: florid counterpoint

`http://strasheela.sourceforge.net/strasheela/doc/Example-FloridCounterpoint.html`

- Context of simultaneous notes constrained, but inaccessible in CSP definition
- CSP defines relatively complex combinatorial problem. Rules which cause particular complexity (together with standard rhythmic, harmonic, and melodic counterpoint rules):
  - Canon
  - Pitch maxima and minima of phrases must differ

Application: Left-to-Right Variable Ordering II

Runtime measurements (full CSP)

- Left-to-right variable ordering: ca. 4 secs (189 distributable spaces, 175 failed spaces, search tree depth 47)
- Distribution which first determines rhythmic structure: no solution after 1 hour!
- Left-to-right variable ordering at least 900 times faster

Application: Left-to-Right Variable Ordering III

Runtime measurements (simplified CSP: no unique maxima and minima pitches required)

- Left-to-right variable ordering: 1.7 secs (92 distributable spaces, 70 failed spaces, search tree depth 53)
- Distribution which first determines rhythmic structure: 14 secs (630 distributable spaces, 601 failed spaces, search tree depth 62)
- Left-to-right variable ordering almost 10 times faster

Application: Left-to-Right Variable Ordering IV

Result

Choice of suitable variable ordering has great influence on efficiency – also in music domain

Recommended Reading I

Computer-aided composition (CAC) in general

- Miranda, E. R (2001). *Composing Music with Computers*. Focal Press. – introduction to CAC in general
- Roads, C. (1996). *The Computer Music Tutorial*. MIT press. – very good survey of whole computer music field, CAC discussed in Chap. 18 “Algorithmic Composition Systems” and 19 “Representation and Strategies for Algorithmic Composition”
- Dodge, C. and Jerse, T. A. (1997). *Computer Music: Synthesis, Composition, and Performance*. Schirmer Books. – sound synthesis textbook with several CAC examples

Recommended Reading II

Music constraint programming

- Pachet, F. and P. Roy (2001). Musical Harmonization with Constraints: A Survey. *Constraints Journal 6*(1). http://www.csl.sony.fr/downloads/papers/2000/pachet-constraints2000.pdf – survey of music constraint programming subfield: harmonisation
- Torsten Anders (2007). *Composing Music by Composing Rules: Design and Usage of a Generic Music Constraint System*. PhD. thesis, Queen's University Belfast. http://strasheela.sourceforge.net/documents/TorstenAnders-PhDThesis.pdf – explains Strasheela in detail, Chap. 3 extensively surveys field music constraint programming

Recommended Reading III

Constraint programming (CP) in general

- Roman Barták (1998). *On-Line Guide to Constraint Programming*. `http://kti.ms.mff.cuni.cz/~bartak/constraints/index.html` – gentle introduction to CP
- Apt, K. R. (2003). *Principles of Constraint Programming*. Cambridge University Press. – general overview of the field with many CSP examples
- Dechter, R. (2003). *Constraint Processing*. Morgan Kaufmann. – explains various constraint solving algorithms

Recommended Reading IV

Constraint programming model used by Strasheela

- van Roy, P. and S. Haridi (2004). *Concepts, Techniques, and Models of Computer Programming*. MIT Press. – highly recommended programming textbook in general (google for computer music textbook), Chap. 12 explains space-based constraint model
- Schulte, C. (2002). *Programming Constraint Services*. Springer-Verlag. – most detailed explanation of the space-based constraint model, advanced text

OpenSound Control Interface for Oz (MSc project) I

- Strasheela results can be exported in various formats for music notation and sound synthesis
- This project will add OpenSound Control output to Strasheela
- OpenSound Control (OSC) is communication protocol used by many music applications
- OSC exceeds the widespread MIDI standard (e.g., more flexibility what data is send, operates at broadband network speeds)
- Project will create an Oz interface for an existing cross-platform OSC library (C or C++ library, e.g., liblo)

OpenSound Control Interface for Oz (MSc project) II

URLS

- Strasheela: `http://strasheela.sourceforge.net`
- OSC: `http://www.cnmat.berkeley.edu/OpenSoundControl/`
- liblo: `http://liblo.sourceforge.net`
- Oz: `http://www.mozart-oz.org`

A Graphical User Interface for Strasheela (MRes project) I

- Strasheela highly expressive composition system
- Its user interface is the programming language Oz: suitable for expert users, but makes learning Strasheela hard for new users
- This project will design and implement a graphical user interface for important Strasheela functionality
- Strasheela is programming system: its interface must allow high degree of flexibility

A Graphical User Interface for Strasheela (MRes project) II

- Possible solution: visual programming language (VPL) – many successful music programming systems with VPL exist
- Possible approaches
  - VPL based on existing VPL system for music (e.g. PWGL or OpenMusic) – generates Strasheela code, communication via socket
  - Design of new VPL, e.g., implemented with QTk, a high-level Tk interface provided by Oz

A Graphical User Interface for Strasheela (MRes project) III

URLS

- Strasheela: `http://strasheela.sourceforge.net`
- PWGL: `http://www2.siba.fi/PWGL/`
- OpenMusic: `http://recherche.ircam.fr/equipes/repmus/OpenMusic/`
- Oz: `http://www.mozart-oz.org`

Summary

- Motivation of problem-specific variable and value orderings
- Constraint model based on computational spaces allows for user-defined and dynamic variable and value orderings
- Score distribution strategies implement problem-specific variable/value orderings for musical CSPs. Examples
  - Common technique in general: first-fail
  - Distribution strategies for resolving inaccessible score contexts
  - Left-to-right variable ordering